Exercise 2C: Applying Permutations

1. How many different 4-digit numbers less than 6 000 can be made from the digits 7, 6, 4 and 2? The digits may not be used more than once.

2. How many different 5-digit numbers can be formed from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9? Digits are to be used once only and zero may not be used in the front of a number.

3. A second-hand car yard has 6 cars and 2 camper-vans to be displayed in a row along its footpath frontage.
   a. How many different arrangements of all 8 vehicles are possible?
   b. How many different arrangements are possible if a camper-van is to be on each end of the display?

4. A swimming squad has 8 swimmers. The first, second, third and fourth swimmers for a relay team must be decided. In how many different ways may this be done if:
   a. there are no restrictions?
   b. one particular swimmer must swim fourth?
   c. also, a fifth swimmer must be named as a reserve?

5. There is a row of 12 parking bays, but only 7 cars are parked in them.
   a. How many different possible arrangements of cars are possible?
   b. If one of the cars belongs to the manager and he always parks in the reserved bay, how many different arrangements of cars are possible?

6. There are 11 men in a cricket team. How many different batting orders of 10 men are possible? Three particular batsmen always bat before any other players. How many different batting orders are possible in this case?

7. How many 3-digit even numbers can be formed with the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if repetition of digits is:
   a. not allowed? b. allowed?

8. How many 6-digit numbers, which are multiples of 5, can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9? Repetitions of digits are not allowed.

9. Four-digit numbers can be made up by permuting all of the digits 5, 6, 1 and 3. Digits may be used only once in each permutation.
   a. List all of these numbers which are greater than 6 000.
   b. How many of these numbers would be less than 6 000?

10. In how many ways can the letters of the word *PERMUTE* be arranged if both *E*s stay in the same place: second and seventh?

11. A crossword fan has one word to complete.

			E				E

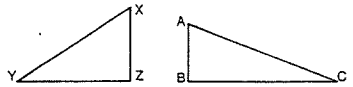
The two Es are fixed in place and she decides that the remaining seven letters must be from the list: S I A M B R L T or N.
   a. How many different ways are there of arranging 7 different letters from the list?
   b. The missing word means "unhappy". What is it?

12. There are 7 hooks in a row on a wall and 5 paintings to hang on them. In how many different ways can the paintings be arranged? How many of these arrangements leave the middle hook vacant?

13. In how many different ways can 5 oil paintings of different value be arranged along a wall if:
   a. the most valuable is placed in the middle of the display?
   b. the two least valuable paintings are placed at the ends of the display?

14. A girl collects model cars and now has 15 of them. She has room on her top shelf for 6 and on her bottom shelf for 5. The rest cannot go on display. How many different arrangements are possible for:
   a. the top shelf? b. the top and bottom shelves?

15. The following two triangles are equal in area.



How many different ways are there of writing this true sentence: "Triangle **XYZ** has the same area as triangle **ABC**", if only the letters in heavy type may be rearranged?

16. In an experiment on short-term memory, subjects are asked to read a list of 10 non-sense syllables. These are presented one at a time on cards.

RAQ	*TES*	*FIL*	*POZ*	*NUR*
BAC	*CEY*	*GIC*	*HUS*	*KOH*

After a short fixed interval of time each each subject is asked to write down all of the nonsense syllables in the correct order.
   a. How many **correct** orders are possible?
   b. How many different cards consisting of a consonant, followed by a vowel, followed by a different consonant could be prepared?

To take away the effect of order of presentation, the syllables are presented to each subject in a different order.
   c. In how many different orders can the cards be presented?
   d. Which two cards do you guess that nearly all subjects recall correctly?

17. The comics page of the paper consists of 5 comic strips and one large advertisement. The comic strips and the advertisement each occupy the full width of the page.
   a. In how many different ways can the page be set out?
   b. In how many different ways can the page be set out if the advertisement must be at the top?
   c. In how many ways can the page be set out if all of the comics must be together?

18. Consider the symbols: *X 8 T I 2 4*
   a. i. For fun, write down a permutation of all of the symbols so that a sentence is formed when they are read from left to right.
      ii. How many permutations of all six symbols are possible?
   b. i. For fun, write down a permutation of four of these symbols so that a sentence is formed when they are read from left to right.
      ii. How many different permutations of **these** four symbols are possible?
   c. A computer programme is designed to print out all of the possible arrangements of any four symbols chosen from this list. How many different arrangements will it print?
   d. i. For fun, arrange these symbols to form a sentence: *M 5 T 2 U X C*
      ii. How many arrangements of these symbols have *M* and *T* together in that order?

19. In how many ways can 7 men and 6 women arrange themselves in a row if men and women must alternate?

20. The mayor, the town clerk and 10 councillors attend a council meeting. In how many different ways may they arrange themselves around the table if:
   a. the mayor always sits at the head of the table?
   b. the mayor always sits at the head of the table and the town clerk sits on his right?
   c. the mayor always sits at the head of the table and the town clerk sits next to him?

21. A photograph of the seven school prefects is to be taken for the school magazine. In how many different ways may the photographer order the seven students if the photograph is to be taken with:

a. all the prefects in a row?
b. four prefects standing and three sitting?
c. the head boy and head girl seated and the remaining five prefects standing in a row behind them?

22. A model train enthusiast has 2 locomotives: a steam locomotive and diesel locomotive; 8 different flat-top wagons; 3 different passenger carriages and a guardsvan. Determine in how many different ways a train may be coupled containing:

a. a steam locomotive, 8 flat-top wagons and a guardsvan.
b. a diesel locomotive, 8 flat-top wagons, 3 passenger carriages and the guardsvan.
c. two locomotives, 3 passenger carriages and the guardsvan.
d. a diesel locomotive in the front, followed by the 8 flat-top wagons and the guardsvan at the end.

23. In how many different ways can 3 men and 4 women sit in a row if:

a. there are no restrictions? b. the men are to sit together?

24. The State Emergency Service has a list of 8 volunteers: Alice, Bob, Carol, David, Ellen, Fred, Gwen and Harry. The secretary is to roster one volunteer to the telephone switchboard for each day of the Easter holiday period: Good Friday, Easter Saturday, Easter Sunday and Easter Monday. No volunteer may be rostered for more than one day.

a. How many different rosters are possible?
b. How many of these rosters would have Alice working on Easter Monday?
c. How many of these rosters would not have Alice working on Easter Monday?
d. How many different rosters are possible if it was decided to have a man and a woman on the switchboard for each day?

25. In a mixed-doubles tennis match, one man with one woman forms a team. How many different mixed-doubles teams are possible if there are:

a. 6 men and 6 women? b. 8 men and 6 women?

26. Five men and four women enter a ballroom dancing championship. How many different pairs (a man and a woman) can be formed? At the last moment, x **extra** women entered the championship and it was calculated that 2 520 different pairs were possible. Find x.

Hint: Find consecutive factors of 2 520.

27. Two pistol-shooting pairs are to be selected from 5 pistol shooters. A first and second shooter must be nominated for each pair. In how many different ways can:

a. an A-pair and a B-pair be selected? b. two pairs be selected?

28. Three sets of twins stand in a row for a photograph. There is a boy and a girl in each set. In how many different ways could they arrange themselves if:

a. there was no restriction? b. each twin pair must not be separated?
c. each twin pair must not be separated and boys and girls must alternate?

29. There are 4 French books, 3 German books and 2 Italian books to be placed on a shelf. In how many different ways can they be arranged if:

a. there are no restrictions?
b. the French books must be on the left, the German books in the middle, and the Italian books on the right?
c. the French books must be together, the German books must be together, and the Italian books must be together?

30. Three married couples attend the theatre. In how many different ways can they seat themselves in a block of six vacant seats if:

a. there are no restrictions? b. each man decides to sit next to his wife?
c. men and women alternate? d. the three men sit together?

31. A computer is programmed to list all of the permutations of the 8 letters of the word *COMPUTER*.

a. How many permutations are there?

How many of these permutations:

b. start with *C*?
c. start with a consonant?
d. start and end with a consonant?
e. end with the letters *ER* in that order?
f. end with the letters *E* and *R* in either order?
g. have the letters *E* and *R* together?

32. Arnold Schoenberg (1874–1951) invented the "tone row" technique for composing music. In these compositions he used all 12 notes of the musical scale:

$$C,\ C\#,\ D,\ E\flat,\ E,\ F,\ F\#,\ G,\ A\flat,\ A,\ B\flat,\ B$$

To start a composition he set down a permutation of all 12 notes: a tone row.

a. How many different tone rows are possible?

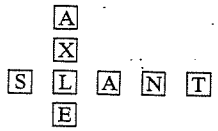
How many of these tone rows:

b. start with *C* and end with $B\flat$?
c. start with *C* and have *C#* second?
d. have *C* followed directly by *C#* somewhere in the row?
e. have *C* and *C#* together somewhere in the row?

33. Words are to be formed from the letters of the word *EDUCATION*. Letters are to be used once only and each word will contain all 9 letters. How many different words:

a. commence with the letter *E* and end with the letter *N*?
b. have the letter *D* following the letter *E*?
c. have the letter *D* adjacent to the letter *E*?
d. have all the vowels together and all the consonants together?
e. have all the vowels together?
f. commence with the letter *T*, followed directly by the letter *E*, and the last two letters *C* and *D* in either order?
g. commence with the letter *A* and end with the letter *A*?

34. In the word game SCRABBLE each player has 7 randomly-selected tiles with letters printed on them. The players must make dictionary words on a playing board by building on letters already in place.



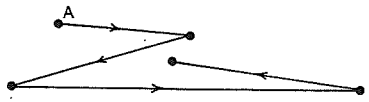
Players must think of as many arrangements of their letters as possible before they make a word. Unfortunately, not every arrangement makes a dictionary word. The first player has [A] [B] [R] [E] [X] [U] [L]. How many different arrangements are possible using:

a. all 7 letters? b. 4 out of the 7 letters?

A second player holds [B] [S] [A] [T] [M] [N] [U] and she decides to build horizontally on the [L]. One possibility is shown in the diagram. How many different arrangements are possible if she decides to:

c. start with [L] and use 4 of her own letters?
d. use the [L] in the second place and use 4 of her own letters?
e. use the [L] in any place and use 4 of her own letters?
f. start with [L] and use 4 of her own letters including her [S] in the last place?

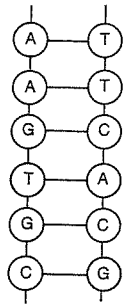
35. The diagram below shows one straight-line pathway connecting 5 dots.



A pathway must start at one dot, end at another and pass through every dot exactly once.

a. How many different pathways are possible?
b. How many different pathways start at dot A?
c. How many different pathways do not start at dot A?
d. How many different pathways end with dot A?
e. How many different pathways have dot A neither at the start nor at the end?
f. It is possible to draw 5 dots with only two different pathways. Draw 5 dots with this property.

36. Deoxyribonucleic acid (DNA) is a long complex molecule found in the nucleus of all living cells. Its structure dictates inherited characteristics: whether a rose will be red, whether a girl will be tall, and so on. The molecule looks like a very long twisted ladder. Each rung of the ladder is called a nucleotide and is made by pairing two bases. There are 4 bases: adenine (A), guanine (G), cytosine (C) and thymine (T). The diagram shows that A can only be paired with T, and G can only be paired with C, and vice versa.



It is the **sequence** of the nucleotides which determines inherited characteristics.

a. How many different sequences of 6 nucleotides are possible? (Assume AT is different from TA and that GC is different from CG).
b. How many different sequences of n nucleotides are possible?
c. What is the minimum number of nucleotides necessary to give 1 000 000 different sequences?

37. How many different 3-permutations can be formed from:

a. a 5-set?
b. a 3-set?
c. i. an n-set? What are the restrictions on n?
ii. Use the information in part i. to find a positive integral root of $n^3 - 3n^2 + 2n - 990 = 0$.

38. Show ${}^{n+1}P_{r+1} - {}^{n}P_{r} = n.\left({}^{n}P_{r}\right)$ where n and r are positive integers and $n \geqslant r$.

39. An integer, r, is greater than 0 but less than 9.

a. How many (r + 2)-permutations are there from a 10-set?
b. There are 12 times more (r + 2)-permutations than r-permutations from a 10-set. Find r.

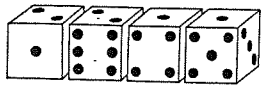
40. A child has a set of 9 blocks. They are identical cubes except that they are all different colours.

a. In how many different orders can she arrange them in a row?
b. If she wishes to make a stack of just 5 of the blocks, how many different arrangements are possible?

41. A red die is placed in front of you on a table so that one face points towards you.

a. i. How many different faces could be uppermost?
ii. With a particular one of these faces uppermost, how many different faces could point towards you?
iii. How many different looking ways could a die be placed on the table so that one of the faces points towards you?

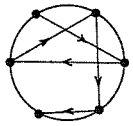
A white die, a green die, and a black die are placed in a row with the red die so that one face of each die points towards you.



b. i. In how many different ways could the colours be permuted?
ii. In how many different looking ways could you arrange the 4 dice in a row?

42. There are 6 points evenly spaced around the circumference of a circle. A straight-line pathway starts at any point and finishes at any other point. Every point is included once in the pathway.

a. How many different pathways are possible?



If the radius of the circle is 1 unit,

b. what is the length of a shortest pathway?
c. how many different shortest pathways are there?

Exercise 2C.

1. 2.3.2.1 = 12
2. 9.9.8.7.6 = 27 216
3. a. 8! = 40 320
   b. 2.6!3 = 1 440
4. a. 8.7.6.5 = 1 680
   b. 7.6.5.1 = 210
   c. 8.7.6.5.4 = 6 720
5. a. ${}^{12}P_7$ = 3 991 680
   b. ${}^{11}P_6$ = 332 640
6. 11! = 39 916 800; 3!8! = 241 920
7. a. 8.7.4 = 224
   b. 9.9.4 = 324
8. 8.7.6.5.4.1 = 6 720
9. a. 6 135, 6 315, 6 513, 6 153, 6 351, 6 531
   b. 4! − 6 = 18
10. 5! = 120
11. a. $\frac{9!}{2!}$ = 181 440
    b. MISERABLE
12. 7.6.5.4.3 = 2 520; 6! = 720
13. a. 4! = 24
    b. 2.3!1 = 12
14. a. ${}^{15}P_6$ = 3 603 600
    b. ${}^{15}P_{11} = \frac{15!}{4!}$
15. 2.3!3! = 72
16. a. 1 b. 21.5.20. = 2 100
    c. 10! = 3 628 800
    d. first and last
17. a. 6! = 720
    b. 1.5! = 120
    c. 2.5! = 240
18. a. i. I82X4T ii. 720
    b. i. I82X ii. 24
    c. 6.5.4.3 = 360
    d. i. UC5MTX2
    ii. 6! = 720
19. 7!6! = 3 628 800
20. a. 11! = 39 916 800
    b. 10! = 3 628 800
    c. 10!2 = 7 257 600
21. a. 5 040 b. 5 040
    c. 2!5! = 240
22. a. 10! = 3 628 800
    b. 13! = 6 227 020 800
    c. 6! = 720
    d. 1.8!1 = 40 320
23. a. 5 040
    b. 3!5! = 6! = 720
24. a. 8.7.6.5 = 1 680
    b. 7.6.5.1 = 210
    c. 1 680 − 210 = 1 470
    d. 4.4.3.3.2.2.1.1 = 576
25. a. 6! = 720
    b. ${}^{6}P_6$ = 20 160
26. 5(4.3.2.1) = 120; x = 3
27. a. 5.4.3.2 = 120
    b. 120 ÷ 2 = 60
28. a. 720 b. 3!2.2.2 = 48
    c. 3!2 = 12
29. a. 362 880
    b. 4!3!2! = 288
    c. 4!3!2!3! = 1 728
30. a. 720 b. 3!2^3 = 48
    c. 2.3!3! = 72
    d. 4.3!3! = 144
31. a. 40 320 b. 1.7! = 5 040
    c. 5.7! = 25 200
    d. 5.4.6! = 14 400
    e. 6!1 = 720
    f. 6!2 = 1 440.
    g. 2.7! = 10 080
32. a. 12! = 479 001 600
    b. 1.10!1 = 3 628 800
    c. 1.1.10! = 3 628 800
    d. 11! = 39 916 800
    e. 2.11! = 79 833 600
33. a. 1.7!1 = 5 040
    b. 8! = 40 320
    c. 2.8! = 80 640
    d. 2.5!4! = 5 760
    e. 5.5!4! = 14 400
    f. 1.1.5!2 = 240
    g. 0
34. a. 5 040 b. 7.6.5.4 = 840
    c. 840 d. 840 e. 4 200
    f. 1.6.5.4.1 = 120
35. a. 5! = 120
    b. 1.4! = 24
    c. 5! − 4! = 96
    d. 4! = 24
    e. 5! − 2(4!) = 72
    f. .....
36. a. 4^6 = 4 096 b. 4^n
    c. $4^9 < 1\,000\,000$, minimum number is 10.
37. a. 60 b. 6
    c. i. n(n − 1)(n − 2) = $n^3 - 3n^2 + 2n$, $n \geqslant 3$ and integral
    ii. n(n − 1)(n − 2) = 990 = 11.10.9, n = 11
39. a. $\frac{10!}{(8-r)!}$ b. r = 6
40. a. 362 880 b. 15 120
41. a. i. 6 ii. 4 iii. 24
    b. i. 4! = 24
    ii. 4!6^4.4^4 = 7 962 624
42. a. 6! = 720 b. 5 units
    c. 2.6 = 12